10/25/17

"Everyone has a gift, some people just never open theirs." -Mr. Callahan

HW: "2017 A2 CC Complex Numbers" TBA

Test 3 on Monday 10/30

AIM: What are Complex Numbers?

Warm Up:

1. $625^{\frac{3}{4}} = \sqrt[4]{625^3}$

$= \boxed{125}$

2. $(3ab^2c)\left(\dfrac{2a^2b}{c^3}\right)^{-1}$

$\dfrac{(3\cancel{a}\cancel{b^2}c)}{1}\left(\dfrac{c^3}{2\cancel{a^2}\cancel{b}}\right)^1 = \dfrac{3bcc^3}{2a}$

$= \boxed{\dfrac{3bc^4}{2a}}$

3. Solve: $x^2 + 1 = 0$

$-1 \quad -1$

$\sqrt{x^2} = \sqrt{-1}$

$\boxed{x = \sqrt{-1}}$

Definition: The imaginary unit, i, is defined as $\sqrt{-1}$. Therefore:

$i^0 = \left(\sqrt{-1}\right)^0 = \boxed{1}$

$i^1 = \boxed{i}$

$i^2 = \sqrt{-1} \cdot \sqrt{-1} = \boxed{-1}$

$i^3 = i^1 \cdot i^2 = i(-1) = \boxed{-i}$

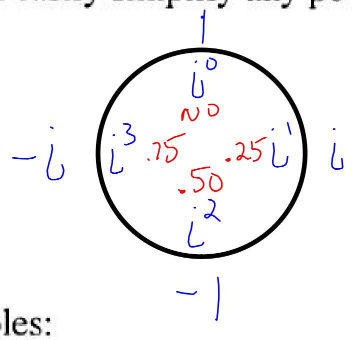
$i^4 = i^2 \cdot i^2 = (-1)(-1) = \boxed{1}$

$i^5 = i^3 i^2 = (-i)(-1) = \boxed{i}$

$i^6 = i^3 \cdot i^3 = (-i)(-i) = i^2 = \boxed{-1}$

$i^7 = i^4 \cdot i^3 = (1)(-i) = \boxed{-i}$

We can easily simplify any power of i. We do this by:



(*) Divide the exponent by 4

no decimal $= i^0 = 1$

$.25 = i^1 = i$

$.5 = i^2 = -1$

$.75 = i^3 = -i$

Examples:

Simplify each.

1. $i^{20} = 1$

$\frac{20}{4} = 5.00$

2. $i^{39} = \boxed{-i}$

$\frac{39}{4} = 9.75$

3. $i^{78} = -1$

$\frac{78}{4} = 19.50$

4. $3i^{11} \cdot 2i^5$

$\frac{11}{4} = 2.75$

$\frac{5}{4} = 1.25$

$3(-i) \cdot 2(i)$

$-3i \cdot 2i$

$-6i^2$

$-6(-1) = \boxed{6}$

Property of negative square roots:

$$\sqrt{-c} = \sqrt{-1c} = \sqrt{-1}\sqrt{c} = i\sqrt{c}$$

pull out negative
as "i"

Examples:

Simplify each.

5. $\sqrt{-25}$

$i\sqrt{25}$

$i \cdot 5$

$\boxed{5i}$

$\sqrt{-25}$

$\sqrt{-1}\ \sqrt{25}$

$i \cdot 5$

$\boxed{5i}$

6. $\sqrt{-32} = \boxed{4i\sqrt{2}}$

$i\sqrt{32}$

$\sqrt{16}\ \sqrt{2}$

$i\,4\sqrt{2}$

7. $-\sqrt{25} - \sqrt{-147}$

$-5 - i\sqrt{147}$

$-5 - i\sqrt{49}\ \sqrt{3}$

$\boxed{-5 - 7i\sqrt{3}}$

8. $\sqrt{-128}$

$\sqrt{64}\ \sqrt{2}$

$i \cdot 8\sqrt{2}$

$\boxed{8i\sqrt{2}}$

9. $\sqrt{-9} + \sqrt{-16}$

$i\sqrt{9} \quad i\sqrt{16}$

$3i + 4i = \boxed{7i}$

Definition:

A number of the form $\boxed{a+bi}$ where a and b are real numbers and $i=\sqrt{-1}$ is called a **complex number.** a is called the **real part** and bi is called the **imaginary part.** A complex number written with the real part first and the imaginary part last is in **standard form.**

Examples:

Perform the operations and put your answers in standard form.

10. $(-1+2i)+(5-3i)$

$-1+5=4$

$2i+(-3i)=-1i$

$\boxed{4-1i}$

11. $(-11-40i)-(2+10i)$

$-11\ -40i\ -2\ -10i$

$\boxed{-13-50i}$

12. $10i(6-8i)$

$60i+80\cancel{i^2}$

$60i-80(-1)$

$60i+80$

$\boxed{80+60i}$

13. $(2+5i)(3-15i)$

	2	+5i	
	6	15i	3
	-30i	+75$\cancel{i^2}$	-15i

6

$-15i$

75

$81-15i$

14. $\sqrt{-4}\cdot\sqrt{-10}\cdot\sqrt{36}$

15. $(5-\sqrt{-27})-(9+\sqrt{-108})$

$5-\sqrt{-27}-9-\sqrt{-108}$

$-i\sqrt{9}\sqrt{3}\qquad -i\sqrt{36}\sqrt{3}$

$-i\,3\sqrt{3}\qquad -i\,6\sqrt{3}$

$5\ -3i\sqrt{3}\ -9\ -6i\sqrt{3}$

$-4-9i\sqrt{3}$

16. $(-2+6i)(3-2i)$

$-6\underbrace{+4i+18i}\ +12\cancel{i^2}$

$\boxed{-6}\ +22i\ \boxed{+12}$

$\boxed{6+22i}$

17. $(4+i)(-5-3i)$

18. Simplify: $5i^{18}+7i^{25}+2i^{28}+6i^{43}$

$\frac{18}{4}=4.5\ \ -1\qquad \frac{28}{4}=7.00\ \ 1$

$\frac{25}{4}=6.25\ \ i\qquad \frac{43}{4}=10.75\ \ -i$

$5(-1)+7i+2(1)+6(-i)$

$-5+7i+2-6i$

$\boxed{-3+i}$

19. Determine the result in simplest $a+bi$ form:

$$(5+2i)(-3+i)+4i(2+3i)$$

Definition:

$a+bi$ and $a-bi$ are called **complex conjugates.** So, $(a+bi)(a-bi)=$

$$(2+5i)(3-15i)$$

$$6-30i+15i+75\cancel{i^2}$$

$$6-30i+15i+75$$

$$81-15i$$